



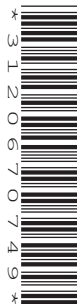
Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

May/June 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.



[5]

[illegible]



- 2 (a)** Starting from the definitions of $\tanh$ and sech in terms of exponentials, prove that

$$\tanh^2 t + \operatorname{sech}^2 t = 1. \quad [3]$$

[illegible]

- (b) The curve C has parametric equations

$$x = \ln(\cosh t), \quad y = \tan^{-1}(\sinh t), \quad \text{for } 0 \leq t \leq 1.$$

Find the length of C . [5]

[illegible]



(b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(4, \frac{1}{3})$.

[5]



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Handwriting practice area with 20 sets of dotted lines.





5 (a) Use de Moivre's theorem to show that

$$\sec 5\theta = \frac{\sec^5 \theta}{5 \sec^4 \theta - 20 \sec^2 \theta + 16}. \quad [6]$$

[illegible]



(b) Hence, obtain the roots of the equation

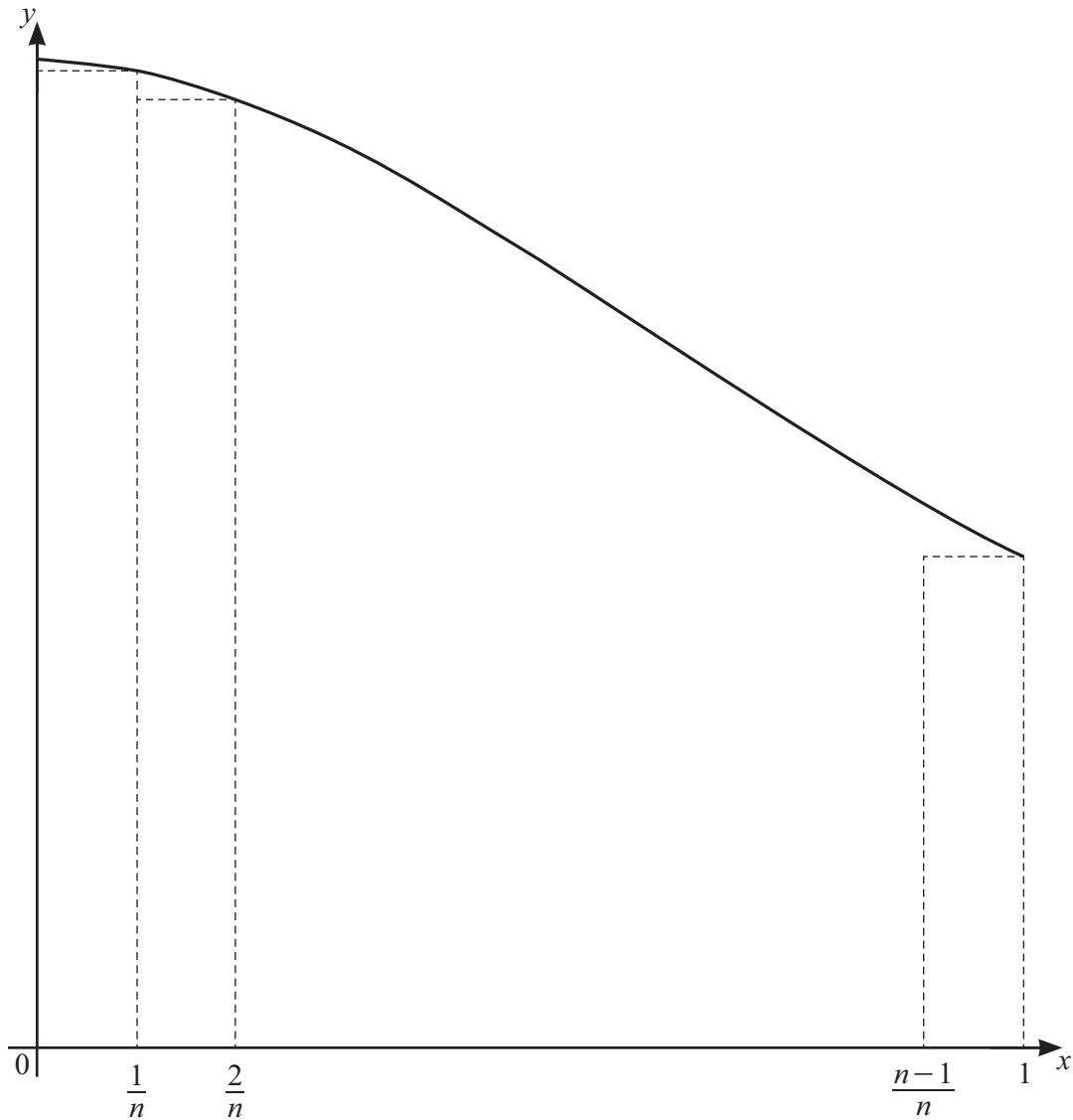
$$\sqrt{3}x^5 - 10x^4 + 40x^2 - 32 = 0$$

in the form $\sec(q\pi)$, where q is rational.

[4]

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6



The diagram shows the curve with equation $y = \frac{1}{x^2 + 1}$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{n}{n^2 + r^2} < \frac{1}{4}\pi. \quad [5]$$

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- (b) Use a similar method to find a lower bound for $\sum_{r=1}^n \frac{n}{n^2 + r^2}$. Give your answer in terms of n and π . [4]

- (c) Deduce the exact value of $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{n}{n^2 + r^2}$. [1]





7 Find the solution of the differential equation

$$\frac{dy}{dx} - \frac{2x+6}{x^2+6x+5}y = 4,$$

given that $y = 0$ when $x = 0$. Give your answer in an exact form.

[9]

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[illegible]



- 8 (a) Find the values of a for which the system of equations

$$\frac{3}{2}x + 3y + 8z = 1,$$

$$ax + 3y + 4z = 2,$$

$$ay - z = 3,$$

does not have a unique solution.

[3]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} \frac{3}{2} & 3 & 8 \\ 0 & 3 & 4 \\ 0 & 0 & -1 \end{pmatrix}.$$

- (b) Given that $\mathbf{B} = \mathbf{A}^{-1}$, use the characteristic equation of $\mathbf{A}$ to show that $\mathbf{B}^2 = p\mathbf{I} + q\mathbf{A}$, where p and q are constants to be determined. [4]

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(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^{-1} = \mathbf{PDP}^{-1}$. [7]

[illegible]



If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

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